

## Lecture Presentation

## Thermodynamics: Entropy, Free Energy, and Equilibrium

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## Spontaneous Processes

## Spontaneous Process: A process that, once started, proceeds on its own without a continuous external influence



## Spontaneous Processes



The rusting of these wheel rims is a slow but spontaneous reaction.


Reaction progress $\qquad$

## Enthalpy, Entropy, and Spontaneous <br> Processes

State Function: A function or property whose value depends only on the present state, or condition, of the system, not on the path used to arrive at that state

Enthalpy Change ( $\Delta \boldsymbol{H}$ ): The heat change in a reaction or process at constant pressure;
$\Delta H=\Delta E+P \Delta V$
Entropy (S): The amount of molecular randomness in a system

## Enthalpy, Entropy, and Spontaneous Processes



## Enthalpy, Entropy, and Spontaneous Processes

Exothermic:
$\mathrm{CH}_{4}(g)+2 \mathrm{O}_{2}(g) \longrightarrow \mathrm{CO}_{2}(g)+2 \mathrm{H}_{2} \mathrm{O}(\mathrm{I}) \quad \Delta H^{\circ}=-890.3 \mathrm{~kJ}$
Endothermic:

$$
\begin{array}{lr}
\mathrm{H}_{2} \mathrm{O}(s) \longrightarrow \mathrm{H}_{2} \mathrm{O}(\Lambda) & \Delta H_{\text {fusion }}=+6.01 \mathrm{~kJ} \\
\mathrm{H}_{2} \mathrm{O}(\Lambda) \longrightarrow \mathrm{H}_{2} \mathrm{O}(g) & \Delta H_{\text {vap }}=+40.7 \mathrm{~kJ} \\
\mathrm{~N}_{2} \mathrm{O}_{4}(g) \longrightarrow 2 \mathrm{NO}_{2}(g) & \Delta H^{\circ}=+55.3 \mathrm{~kJ} \\
\mathrm{NaCl}(s) \xrightarrow{\mathrm{H}_{2} \mathrm{O}} \mathrm{Na}+(a q)+\mathrm{Cl}-(\mathrm{aq}) & \Delta H^{\circ}=+3.88 \mathrm{~kJ}
\end{array}
$$

## Enthalpy, Entropy, and Spontaneous Processes



## Enthalpy, Entropy, and Spontaneous Processes



## Enthalpy, Entropy, and Spontaneous Processes



Disruption of the crystal increases the entropy, but the hydration process decreases the entropy. For the dissolution of NaCl , the net effect is an entropy increase.

## Entropy and Probability



## Entropy and Probability

(a) The perfectly ordered
"heads-up" structure.


20 "heads"<br>0 "tails"

(b) The molecules arranged randomly in one of the $2^{20}$ ways in which a disordered structure can be obtained.


## $S=k \ln W$

$k=$ Boltzmann's constant. $=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$

## $W=$ The number of ways that the state can be achieved.

9 "heads"
11 "tails"

## Entropy and Temperature

## Third Law of Thermodynamics: The entropy of a perfectly ordered crystalline substance at 0 K is zero.



Lower temperature:
-Lesser molecular speeds

- Narrower distribution of individual
kinetic energies
- Less randomness
- Lower entropy


## Entropy and Temperature



## Standard Molar Entropies and Standard Entropies of Reaction

## Standard Molar Entropy ( $\mathbf{S}^{\circ}$ : The entropy of 1 mole of a pure substance at 1 atm pressure and a specified temperature.

TABLE 17.1 Standard Molar Entropies for Some Common Substances at $25^{\circ} \mathrm{C}$

| Substance | Formula | $S^{\circ}[\mathbf{J} /(\mathbf{K} \cdot \mathbf{m o l})]$ | Substance | Formula | $S^{\circ}[\mathbf{J} /(\mathbf{K} \cdot \mathbf{m o l})]$ |
| :--- | :--- | :--- | :--- | :--- | :---: |
| Gases |  |  | Liquids |  |  |
| Acetylene | $\mathrm{C}_{2} \mathrm{H}_{2}$ | 200.8 | Acetic acid | $\mathrm{CH}_{3} \mathrm{CO}_{2} \mathrm{H}$ | 160 |
| Ammonia | $\mathrm{NH}_{3}$ | 192.3 | Ethanol | $\mathrm{CH}_{3} \mathrm{CH} \mathrm{H}_{2} \mathrm{OH}$ | 161 |
| Carbon dioxide | $\mathrm{CO}_{2}$ | 213.6 | Methanol | $\mathrm{CH}_{3} \mathrm{OH}$ | 127 |
| Carbon monoxide | CO | 197.6 | Water | $\mathrm{H}_{2} \mathrm{O}$ | 69.9 |
| Ethylene | $\mathrm{C}_{2} \mathrm{H}_{4}$ | 219.5 | Solids |  |  |
| Hydrogen | $\mathrm{H}_{2}$ | 130.6 | Calcium carbonate | $\mathrm{CaCO}_{3}$ | 91.7 |
| Methane | $\mathrm{CH}_{4}$ | 186.2 | Calcium oxide | CaO | 38.1 |
| Nitrogen | $\mathrm{N}_{2}$ | 191.5 | Diamond | C | 2.4 |
| Nitrogen dioxide | $\mathrm{NO}_{2}$ | 240.0 | Graphite | C | 5.7 |
| Dinitrogen tetroxide | $\mathrm{N}_{2} \mathrm{O}_{4}$ | 304.3 | Iron | $\mathrm{Fe}^{\circ}$ | 27.3 |
| Oxygen | $\mathrm{O}_{2}$ | 205.0 | Iron(III) oxide | $\mathrm{Fe}_{2} \mathrm{O}_{3}$ | 87.4 |

# Standard Molar Entropies and Standard Entropies of Reaction 

$$
\begin{gathered}
\Delta S^{\circ}=S^{\circ}(\text { products })-S^{\circ}(\text { reactants }) \\
a \mathrm{~A}+b \mathrm{~B} \longrightarrow c \mathrm{C}+d \mathrm{D} \\
\Delta S^{\circ}=\underbrace{\left[c S^{\circ}(\mathrm{C})+d S^{\circ}(\mathrm{D})\right]}_{\text {Products }}-\underbrace{\left[a S^{\circ}(\mathrm{A})+b S^{\circ}(\mathrm{B})\right]}_{\text {Reactants }}
\end{gathered}
$$

## Standard Molar Entropies and Standard Entropies of Reaction

Using standard entropies, calculate the standard entropy change for the decomposition of $\mathrm{N}_{2} \mathrm{O}_{4}$.

$$
\begin{aligned}
& \mathrm{N}_{2} \mathrm{O}_{4}(g) \longrightarrow 2 \mathrm{NO}_{2}(g) \\
& \Delta S^{\circ}=2 S^{\circ}\left(\mathrm{NO}_{2}(g)\right)-S^{\circ}\left(\mathrm{N}_{2} \mathrm{O}_{4}(g)\right) \\
& \Delta S^{\circ}=(2 \mathrm{~mol})\left(240.0 \frac{\mathrm{~J}}{\mathrm{~K} \mathrm{~mol}}\right)-(1 \mathrm{~mol})\left(304.3 \frac{\mathrm{~J}}{\mathrm{~K} \mathrm{~mol}}\right) \\
& \Delta S^{\circ}=175.7 \mathrm{~J} / \mathrm{K}
\end{aligned}
$$

## Entropy and the Second Law of Thermodynamics

First Law of Thermodynamics: In any process, spontaneous or nonspontaneous, the total energy of a system and its surroundings is constant.

Second Law of Thermodynamics: In any spontaneous process, the total entropy of a system and its surroundings always increases.

## Entropy and the Second Law of Thermodynamics

$$
\begin{aligned}
& \Delta S_{\text {total }}=\Delta S_{\text {system }}+\Delta S_{\text {surroundings }} \\
& \quad \text { or } \\
& \Delta S_{\text {totaa }}=\Delta S_{\text {sys }}+\Delta S_{\text {surr }}
\end{aligned}
$$

$\Delta S_{\text {total }}>0$ The reaction is spontaneous.
$\Delta S_{\text {total }}<0$ The reaction is nonspontaneous.
$\Delta S_{\text {total }}=0 \quad$ The reaction mixture is at equilibrium.

## Entropy and the Second Law of Thermodynamics


(a) When an exothermic reaction occurs in the system ( $\Delta H<0$ ), the surroundings gains heat and its entropy increases ( $\Delta S_{\text {surr }}>0$ ).

(b) When an endothermic reaction occurs in the system ( $\Delta H>0$ ), the surroundings loses heat and its entropy decreases $\left(\Delta S_{\text {surr }}<0\right)$.
$\Delta S_{\text {surr }} \alpha-\Delta H$

$\Delta S_{\text {surf }}=\frac{-\Delta H}{T}$

## Free Energy

Free Energy: $G=H-T S$

$$
\Delta G=\Delta H-T \Delta S
$$

$$
\text { Using: } \Delta S_{\text {total }}=\Delta S_{\text {sys }}+\Delta S_{\text {surr }}
$$

$$
\Delta S_{\text {surr }}=\frac{-\Delta H}{T}
$$

$$
\Delta S=\Delta S_{\mathrm{sys}}
$$

$$
\Delta G=\Delta H-T \Delta S=-T \Delta S_{\text {total }}
$$

Free Energy

Using the second law and $\Delta G=\Delta H-T \Delta S=-T \Delta S_{\text {total }}$
$\Delta \boldsymbol{G}<\mathbf{0} \quad$ The reaction is spontaneous.
$\Delta G>0$
The reaction is nonspontaneous.
$\Delta \boldsymbol{G}=\mathbf{0} \quad$ The reaction mixture is at equilibrium.

## Free Energy

## TABLE 17.2 Signs of Enthalpy, Entropy, and Free-Energy Changes and Reaction Spontaneity for a

 Reaction at Constant Temperature and Pressure| $\Delta H$ | $\Delta S$ | $\Delta G=\Delta H-T \Delta S$ | Reaction Spontaneity | Example |
| :---: | :---: | :---: | :---: | :---: |
| - | + | - | Spontaneous at all temperatures | $2 \mathrm{NO}_{2}(g) \longrightarrow \mathrm{N}_{2}(g)+2 \mathrm{O}_{2}(g)$ |
| + | - | + | Nonspontaneous at all temperatures | $3 \mathrm{O}_{2}(\mathrm{~g}) \longrightarrow 2 \mathrm{O}_{3}(\mathrm{~g})$ |
| - | - | - or + | Spontaneous at low temperatures where $\Delta H$ outweighs $T \Delta S$ | $\mathrm{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g}) \longrightarrow 2 \mathrm{NH}_{3}(\mathrm{~g})$ |
|  |  |  | Nonspontaneous at high temperatures where $T \Delta S$ outweighs $\Delta H$ |  |
| + | + | - or + | Spontaneous at high temperatures where $T \Delta S$ outweighs $\Delta H$ | $2 \mathrm{HgO}(s) \longrightarrow 2 \mathrm{Hg}(l)+\mathrm{O}_{2}(g)$ |
|  |  |  | Nonspontaneous at low temperatures where $\Delta H$ outweighs $T \Delta S$ |  |

## Standard Free-Energy Changes for Reactions

Thermodynamic Standard State: Most stable form of a substance at 1 atm pressure and at a specified temperature, usually $25^{\circ} \mathrm{C} ; 1 \mathrm{M}$ concentration for all substances in solution

$$
\Delta G^{\circ}=\Delta H^{\circ}-T \Delta S^{\circ}
$$

## Standard Free-Energy Changes for Reactions

Calculate the standard free-energy change at $25^{\circ} \mathrm{C}$ for the Haber synthesis of ammonia using the given values for the standard enthalpy and standard entropy changes:

$$
\begin{gathered}
\mathrm{N}_{2}(g)+3 \mathrm{H}_{2}(g) \longrightarrow 2 \mathrm{NH}_{3}(g) \Delta H^{\circ}=-92.2 \mathrm{~kJ} \\
\Delta S^{\circ}=-198.7 \mathrm{~J} / \mathrm{K} \\
\Delta G^{\circ}=\Delta H^{\circ}-T \Delta S^{\circ} \\
\Delta G^{\circ}=-92.2 \mathrm{~kJ}-\left(298 \mathrm{~K} \times \frac{-198.7 \mathrm{~J}}{\mathrm{~K}} \times \frac{1 \mathrm{~kJ}}{1000 \mathrm{~J}}\right)=-33.0 \mathrm{~kJ}
\end{gathered}
$$

## Standard Free Energies of Formation

$$
\Delta G^{\circ}=\Delta G_{f}^{\circ}(\text { products })-\Delta G_{f}^{\circ}(\text { reactants })
$$

$$
a \mathrm{~A}+b \mathrm{~B} \longrightarrow c \mathrm{C}+d \mathrm{D}
$$

## $\Delta G^{\circ}=\underbrace{\left[c \Delta G^{\circ}{ }_{f}(C)+d \Delta G^{\circ}(D)\right]}-\underbrace{\left[a \Delta G_{f}^{\circ}(A)+b \Delta G^{\circ}(B)\right]}$ <br> Products

## Standard Free Energies of Formation

TABLE 17.3 Standard Free Energies of Formation for Some Common Substances at $25^{\circ} \mathrm{C}$

| Substance | Formula | $\Delta \boldsymbol{G}_{\mathbf{f}}{ }^{\circ}[\mathbf{k J} / \mathbf{m o l}]$ | Substance | Formula | $\boldsymbol{\Delta} \boldsymbol{G}_{\mathrm{f}}{ }^{\circ}[\mathbf{k J} / \mathbf{m o l}]$ |
| :--- | :--- | :---: | :--- | :--- | :--- |
| Gases |  |  | Liquids |  |  |
| Acetylene | $\mathrm{C}_{2} \mathrm{H}_{2}$ | 209.9 | Acetic acid | $\mathrm{CH}_{3} \mathrm{CO}_{2} \mathrm{H}$ | -390 |
| Ammonia | $\mathrm{NH}_{3}$ | -16.5 | Ethanol | $\mathrm{CH}_{3} \mathrm{CH}_{2} \mathrm{OH}$ | -174.9 |
| Carbon dioxide | $\mathrm{CO}_{2}$ | -394.4 | Methanol | $\mathrm{CH}_{3} \mathrm{OH}$ | -166.6 |
| Carbon monoxide | CO | -137.2 | Water | $\mathrm{H}_{2} \mathrm{O}$ | -237.2 |
| Ethylene | $\mathrm{C}_{2} \mathrm{H}_{4}$ | 68.1 | Solids |  |  |
| Hydrogen | $\mathrm{H}_{2}$ | 0 | Calcium carbonate | $\mathrm{CaCO}_{3}$ | -1129.1 |
| Methane | $\mathrm{CH}_{4}$ | -50.8 | Calcium oxide | CaO | -603.3 |
| Nitrogen | $\mathrm{N}_{2}$ | 0 | Diamond | C | 2.9 |
| Nitrogen dioxide | $\mathrm{NO}_{2}$ | 51.3 | Graphite | C | 0 |
| Dinitrogen tetroxide | $\mathrm{N}_{2} \mathrm{O}_{4}$ | 99.8 | Iron(III) oxide | $\mathrm{Fe}_{2} \mathrm{O}_{3}$ | -742.2 |

## Standard Free Energies of Formation

Using table values, calculate the standard free-energy change at $25^{\circ} \mathrm{C}$ for the reduction of iron(III) oxide with carbon monoxide:

$$
\begin{aligned}
& \mathrm{Fe}_{2} \mathrm{O}_{3}(s)+3 \mathrm{CO}(g) \longrightarrow 2 \mathrm{Fe}(\mathrm{~s})+3 \mathrm{CO}_{2}(g) \\
& \Delta G^{\circ}= {\left[2 \Delta \mathrm{G}_{\mathrm{f}}^{\circ}(\mathrm{Fe}(s))+3 \Delta \mathrm{G}_{\mathrm{f}}^{\circ}\left(\mathrm{CO}_{2}(g)\right)\right] } \\
&-\left[1 \Delta \mathrm{G}_{\mathrm{f}}^{\circ}\left(\mathrm{Fe}_{2} \mathrm{O}_{3}(s)\right)+3 \Delta \mathrm{G}_{\mathrm{f}}^{\circ}(\mathrm{CO}(\mathrm{~g}))\right] \\
& \Delta G^{\circ}= {[(2 \mathrm{~mol})(0 \mathrm{~kJ} / \mathrm{mol})+(3 \mathrm{~mol})(-394.4 \mathrm{~kJ} / \mathrm{mol})] } \\
& \Delta G^{\circ}=-[(1 \mathrm{~mol})(-742.2 \mathrm{~kJ} / \mathrm{mol})+(3 \mathrm{~mol})(-137.2 \mathrm{~kJ} / \mathrm{mol})]
\end{aligned}
$$

Free Energy Changes and the Reaction Mixture

$$
\Delta G=\Delta G^{\circ}+R T \ln Q
$$

$\Delta G=$ Free-energy change under nonstandard conditions.

For the Haber synthesis of ammonia:

$$
\mathrm{N}_{2}(g)+3 \mathrm{H}_{2}(g) \rightleftharpoons 2 \mathrm{NH}_{3}(g) \quad Q_{\mathrm{p}}=\frac{\left(P_{\mathrm{NH}_{3}}\right)^{2}}{\left(P_{\mathrm{N}_{2}}\right)\left(P_{\mathrm{H}_{2}}\right)^{3}}
$$

## Free Energy Changes and the Reaction Mixture

Calculate $\Delta G$ for the formation of ethylene $\left(\mathrm{C}_{2} \mathrm{H}_{4}\right)$ from carbon and hydrogen at $25^{\circ} \mathrm{C}$ when the partial pressures are $100 \mathrm{~atm} \mathrm{H}_{2}$ and $0.10 \mathrm{~atm} \mathrm{C}_{2} \mathrm{H}_{4}$.

$$
2 \mathrm{C}(s)+2 \mathrm{H}_{2}(g) \rightleftharpoons \mathrm{C}_{2} \mathrm{H}_{4}(g) \quad \Delta G^{\circ}=68.1 \mathrm{~kJ}
$$

Use:

$$
\Delta G=\Delta G^{\circ}+R T \ln Q
$$

$$
Q_{\mathrm{p}}=\frac{P_{\mathrm{C}_{2} \mathrm{H}_{4}}}{\left(P_{\mathrm{H}_{2}}\right)^{2}}
$$

Free Energy Changes and the Reaction Mixture

## Calculate In $Q$ :

$$
\ln \left(\frac{0.10}{1002}\right)=-11.51
$$

Calculate DG:
$\Delta G=\Delta G^{\circ}+R T \ln Q$
$=68.1 \mathrm{~kJ} / \mathrm{mol}+\left(8.314 \frac{\mathrm{~J}}{\mathrm{~K} \mathrm{~mol}}\right)(298 \mathrm{~K})(-11.51)\left(\frac{1 \mathrm{~kJ}}{1000 \mathrm{~J}}\right)$
$\Delta G=39.6 \mathrm{~kJ} / \mathrm{mol}$

## Free Energy and Chemical Equilibrium

$$
\Delta G=\Delta G^{\circ}+R T \ln Q
$$

- When the reaction mixture is mostly reactants:

$$
Q \ll 1 \quad R T \ln Q \ll 0 \quad \Delta G<0
$$

The total free energy decreases as the reaction proceeds spontaneously in the forward direction.

- When the reaction mixture is mostly products:

$$
Q \gg 1 \quad R T \ln Q \gg 0 \quad \Delta G>0
$$

The total free energy decreases as the reaction proceeds spontaneously in the reverse direction.

## Free Energy and Chemical Equilibrium

Beginning with pure reactants, the free energy decreases $(\Delta G<0)$ as the system moves toward equilibrium.

Beginning with pure products, the free energy also decreases ( $\Delta G<0$ ) as the system moves toward equilibrium.


## Free Energy and Chemical Equilibrium

$$
\begin{gathered}
\qquad \Delta G=\Delta G^{\circ}+R T \ln Q \\
\text { At equilibrium, } \Delta G=0 \text { and } Q=K .
\end{gathered}
$$

$$
\Delta G^{\circ}=-R T \ln K
$$

TABLE 17.4 Relationship Between the Standard Free-Energy Change and the Equilibrium Constant for a Reaction: $\Delta \mathbf{G}^{\circ}=-R T \ln K$

| $\boldsymbol{\Delta} \boldsymbol{G}^{\circ}$ | $\ln \boldsymbol{K}$ | $\boldsymbol{K}$ | Comment |
| :---: | :---: | :---: | :--- |
| $\Delta G^{\circ}<0$ | $\ln K>0$ | $K>1$ | The equilibrium mixture is mainly products. |
| $\Delta G^{\circ}>0$ | $\ln K<0$ | $K<1$ | The equilibrium mixture is mainly reactants. |
| $\Delta G^{\circ}=0$ | $\ln K=0$ | $K=1$ | The equilibrium mixture contains comparable |
|  |  |  | amounts of reactants and products. |

## Free Energy and Chemical Equilibrium

Calculate $K_{\mathrm{p}}$ at $25^{\circ} \mathrm{C}$ for the following reaction:

$$
\mathrm{CaCO}_{3}(s) \rightleftharpoons \mathrm{CaO}(s)+\mathrm{CO}_{2}(g)
$$

## Calculate $\Delta G^{\circ}$ :

$$
\begin{aligned}
\Delta G^{\circ}= & {\left[\Delta \mathrm{G}_{\mathrm{f}}^{\circ}(\mathrm{CaO}(s))+\Delta \mathrm{G}_{\mathrm{f}}^{\circ}\left(\mathrm{CO}_{2}(g)\right)\right]-\left[\Delta \mathrm{G}_{\mathrm{f}}^{\circ}\left(\mathrm{CaCO}_{3}(\mathrm{~s})\right)\right] } \\
= & {[(1 \mathrm{~mol})(-603.3 \mathrm{~kJ} / \mathrm{mol})+(1 \mathrm{~mol})(-394.4 \mathrm{~kJ} / \mathrm{mol})] } \\
& -[(1 \mathrm{~mol})(-1129.1 \mathrm{~kJ} / \mathrm{mol})] \\
\Delta G^{\circ}= & +131.4 \mathrm{~kJ} / \mathrm{mol}
\end{aligned}
$$

## Free Energy and Chemical Equilibrium

Calculate In $K$ :

$$
\begin{aligned}
& \Delta G^{\circ}=-R T \ln K \\
& \ln K=\frac{-\Delta G^{\circ}}{R T}=\frac{-131.4 \mathrm{~kJ} / \mathrm{mol}}{\left(8.314 \frac{\mathrm{~J}}{\mathrm{~K} \mathrm{~mol}}\right)(298 \mathrm{~K})\left(\frac{1 \mathrm{~kJ}}{1000 \mathrm{~J}}\right)} \\
& \ln K=-53.04
\end{aligned}
$$

Calculate $K$ :

$$
K=e^{-53.04}=9.2 \times 10^{-24}
$$

