Counting Significant Figures

The precision of a measurement—which depends on the instrument used to make the measurement—must be preserved, not only when recording the measurement, but also when performing calculations that use the measurement. We can accomplish the preservation of this precision by using *significant figures.* In any reported measurement, the non-place-holding digits—those that are not simply marking the decimal place—are called **significant figures** (or **significant digits**). *The greater the number of significant figures, the greater the certainty of the measurement.* For example, the number 23.5 has three significant figures while the number 23.56 has four. To determine the number of significant figures in a number containing zeroes, we must distinguish between zeroes that are significant and those that simply mark the decimal place. For example, in the number 0.0008, the leading zeroes (zeroes to the left of the first non-zero digit) mark the decimal place but *do not* add to the certainty of the measurement and are therefore not significant; this number has only one significant figure. In contrast, the trailing zeroes (zeroes at the end of a number) in the number 0.000800 *do add* to the certainty of the measurement and are therefore counted as significant; this number has three significant figures.

To determine the number of significant figures in a number, follow these rules. (See the examples on the right).

Significant Figure Rules	Examples	
1. All nonzero digits are significant.	28.03	0.0540
2. Interior zeroes (zeroes between two nonzero digits) are significant.	408	7.0301
3. Leading zeroes (zeroes to the left of the first nonzero digit) are not significant. They only serve to locate the decimal point.	0.0032 (0.00 not significant)	0.00006 (0.0000 not significant)
4. Trailing zeroes (zeroes at the end of a number) are categorized as follows:		
• Trailing zeroes after a decimal point are always significant.	45.000	3.5600
• Trailing zeroes before a decimal point (and after a nonzero number) are always significant.	140.00	2500.55
• Trailing zeroes before an *implied* decimal point are ambiguous and should be avoided by using scientific notation.	1200	ambiguous
	1.2×10^3	2 significant figures
	1.20×10^3	3 significant figures
	1.200×10^3	4 significant figures
• Some textbooks put a decimal point after one or more trailing zeroes if the zeroes are to be considered significant. We avoid that practice in this book, but you should be aware of it.	1200.	4 significant figures (common in some textbooks)

Exact Numbers

Exact numbers have no uncertainty and thus do not limit the number of significant figures in any calculation. We can regard an exact number as having an unlimited number of significant figures. Exact numbers originate from three sources:

- From the accurate counting of discrete objects. For example, 3 atoms means 3.00000.... atoms.
- From defined quantities, such as the number of centimeters in 1 m. Because 100 cm is defined as 1 m,

$$100 \text{ cm} = 1 \text{ m means } 100.00000\ldots \text{ cm} = 1.0000000\ldots \text{ m}$$

Significant Figures in Calculations

When we use measured quantities in calculations, the results of the calculation must reflect the precision of the measured quantities. We should not lose or gain precision during mathematical operations. Follow these rules when carrying significant figures through calculations.

Rules for Calculations	Examples
1. In multiplication or division, the result carries the same number of significant figures as the factor with the fewest significant figures.	1.052 (4 sig. figures) × 12.504 (5 sig. figures) × 0.53 (2 sig. figures) = 6.7208 = 6.7 (2 sig. figures) 2.0035 (5 sig. figures) ÷ 3.20 (3 sig. figures) = 0.626094 = 0.626 (3 sig. figures)

Rules for Calculations	Examples
2. In addition or subtraction, the result carries the same number of decimal places as the quantity with the fewest decimal places.	2.345 + 0.07 + 2.9975 = 5.4125 = 5.41 5.9 − 0.221 = 5.679 = 5.7 In addition and subtraction, it is helpful to draw a line next to the number with the fewest decimal places. This line determines the number of decimal places in the answer.
3. When rounding to the correct number of significant figures, round down if the last (or leftmost) digit dropped is four or less; round up if the last (or leftmost) digit dropped is five or more.	Rounding to two significant figures: 5.37 rounds to 5.4 5.34 rounds to 5.3 5.35 rounds to 5.4 5.349 rounds to 5.3 Notice in the last example that only the *last (or leftmost) digit being dropped* determines in which direction to round—ignore all digits to the right of it.
4. To avoid rounding errors in multistep calculations, round only the final answer—do not round intermediate steps. If you write down intermediate answers, keep track of significant figures by underlining the least significant digit.	$6.78 \times 5.903 \times (5.489 - 5.01)$ $= 6.78 \times 5.903 \times 0.479$ $= 19.1707$ $= 19$ underline least significant digit

A few books recommend a slightly different rounding procedure for cases in which the last digit is 5. However, the procedure presented here is consistent with electronic calculators and will be used throughout this book.

Notice that for multiplication or division, the quantity with the fewest *significant figures* determines the number of *significant figures* in the answer, but for addition and subtraction, the quantity with the fewest *decimal places* determines the number of *decimal places* in the answer. In multiplication and division, we focus on significant figures, but in addition and subtraction we focus on decimal places. When a problem involves addition or subtraction, the answer may have a different number of significant figures than the initial quantities. Keep this in mind in problems that involve both addition or subtraction and multiplication or division. For example,

$$\frac{1.002 - 0.999}{3.754} = \frac{0.003}{3.754} = 7.99 \times 10^{-4} = 8 \times 10^{-4}$$

The answer has only one significant figure, even though the initial numbers had three or four.